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On the statistical treatment of football numbers

Frederic M. Lord

Professor X sold "football numbers." The television audience had to have some way to tell which player it was who caught the forward pass. So each player had to wear a number on his football uniform. It didn't matter what number, just so long as it wasn't more than a two-digit number.

Professor X loved numbers. Before retiring from teaching, Professor X had been chairman of the Department of Psychometrics. He would administer tests to all his students at every possible opportunity. He could hardly wait until the tests were scored. He would quickly stuff the scores in his pockets and hurry back to his office where he would lock the door, take the scores out again, add them up, and then calculate means and standard deviations for hours on end.

Professor X locked his door so that none of his students would catch him in his folly. He taught his students very carefully: "Test scores are ordinal numbers, not cardinal numbers. Ordinal numbers cannot be added. *A fortiori,* test scores cannot be multiplied or squared." The professor required his students to read the most up-to-date references on the theory of measurement (e.g., Coombs, 1951; Stevens, 1951; Weitzenhoffer, 1951). Even the poorest student would quickly explain that it was wrong to compute means or standard deviations of test scores.

When the continual reproaches of conscience finally brought about a nervous breakdown, Professor X retired. In appreciation of his careful teaching, the university gave him the "football numbers" concession, together with a large supply of cloth numbers and a vending machine to sell them.

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The first thing the professor did was to make a list of all the numbers given to him. The University had been generous and he found that he had exactly 100,000,000,000,000,000 two-digit cloth numbers to start out with. When he had listed them all on sheets of tabulating paper, he shuffled the pieces of cloth for two whole weeks. Then he put them in the vending machine.

If the numbers had been ordinal numbers, the Professor would have been sorely tempted to add them up, to square them, and to compute means and standard deviations. But these were not even serial numbers; they were only "football numbers"—they might as well have been letters of the alphabet. For instance, there were 2,681,793,401,686,191 pieces of cloth bearing the number "69," but there were only six pieces of cloth bearing the number "68," etc., etc. The numbers were for designation purposes only; there was no sense to them.

The first week, while the sophomore team bought its numbers, everything went fine. The second week the freshman team bought its numbers. By the end of the week there was trouble. Information secretly reached the professor that the numbers in the machine had been tampered with in some unspecified fashion.

The professor barely had time to decide to investigate when the freshman team appeared in a body to complain. They said they had bought 1,600 numbers from the machine, and they complained that the numbers were too low. The sophomore team was laughing at them because they had such low numbers. The freshmen were all for routing the sophomores out of their beds one by one and throwing them in the river.

Alarmed at this possibility, the professor temporized and persuaded the freshmen to wait while he consulted the statistician who lived across the street. Perhaps, after all, the freshmen had gotten low numbers just by chance. Hastily he put on his bowler hat, took his tabulating sheets, and knocked on the door of the statistician.

Now the statistician knew the story of the poor professor's resignation from his teaching. So, when the problem had been explained to him, the statistician chose not to use the elegant nonparametric methods of modern statistical analysis. Instead he took the professor's list of the 100 quadrillion "football numbers" that had been put into the machine. He added them all together and divided by 100 quadrillion.

"The population mean," he said, "is 54.3."

"But these numbers are not cardinal numbers," the professor expostulated. "You can't add them."

"Oh, can't I?" said the statistician. "I just did. Furthermore, after squaring each number, adding the squares, and proceeding in the usual fashion, I find the population standard deviation to be exactly 16.0."

"But you can't multiply 'football numbers,' " the professor wailed. "Why, they aren't even ordinal numbers, like test scores."

"The numbers don't know that," said the statistician. "Since the numbers don't remember where they came from, they always behave just the same way, regardless."

The professor gasped.

"Now the 1,600 'football numbers' the freshmen bought have a mean of 50.3," the statistician continued. "When I divide the difference between population and sample means by the population standard deviation. . . ."

"Divide!" moaned the professor.

". . . And then multiply by $\sqrt{1{,}600}$, I find a critical ratio of 10," the statistician went on, ignoring the interruption. "Now, if your population of 'football numbers' had happened to have a normal frequency distribution, I would be able rigorously to assure you that the sample of 1,600 obtained by the freshmen could have arisen from random sampling only once in 65,618,050,000,000,000,000,000 times; for in this case these numbers obviously would obey all the rules that apply to sampling from any normal population."

"You cannot . . ." began the professor.

"Since the population is obviously not normal, it will in this case suffice to use Tchebycheff's inequality,"[1] the statistician continued calmly. "The probability of obtaining a value of 10 for such a critical ratio in random sampling from any population whatsoever is always less than .01. It is therefore highly implausible that the numbers obtained by the freshmen were actually a random sample of all numbers put into the machine."

"You cannot add and multiply any numbers except cardinal numbers," said the professor.

"If you doubt my conclusions," the statistician said coldly as he showed the professor to the door, "I suggest you try and see how often you can get a sample of 1,600 numbers from your machine with a mean below 50.3 or above 58.3. Good night."

To date, after reshuffling the numbers, the professor has drawn (with replacement) a little over 1,000,000,000 samples of 1,600 from his machine. Of these, only two samples have had means below 50.3 or above 58.3. He is continuing his sampling, since he enjoys the computations. But he has put a lock on his machine so that the sophomores cannot tamper with the numbers again. He is happy because, when he has added together a sample of 1,600 "football numbers," he finds that the resulting sum obey the same laws of sampling as they would if they were real honest-to-God cardinal numbers.

Next year, he thinks, he will arrange things so that the population

[1] Tchebycheff's inequality, in a convenient variant, states that in random sampling the probability that a critical ratio of the type calculated here will exceed any chosen constant, c, is always less than $1/c^2$, irrespective of the shape of the population distribution. It is impossible to devise a set of numbers for which this inequality will not hold.

distribution of his "football numbers" is approximately normal. Then the means and standard deviations that he calculates from these numbers will obey the usual mathematical relations that have been proven to be applicable to random samples from any normal population.

The following year, recovering from his nervous breakdown, Professor X will give up the "football numbers" concession and resume his teaching. He will no longer lock his door when he computes the means and standard deviations of test scores.

References

Coombs, C. H. Mathematical models in psychological scaling. *J. Amer. Stat. Ass.*, 1951, **46**, 480–489.

Stevens, S. S. Mathematics, measurement, and psychophysics. In S. S. Stevens (Ed.), *Handbook of experimental psychology*. New York: Wiley, 1951. Pp. 1–49.

Weitzenhoffer, A. M. Mathematical structures and psychological measurements. *Psychometrika*, 1951, **16**, 387–406.